Physics 2CL

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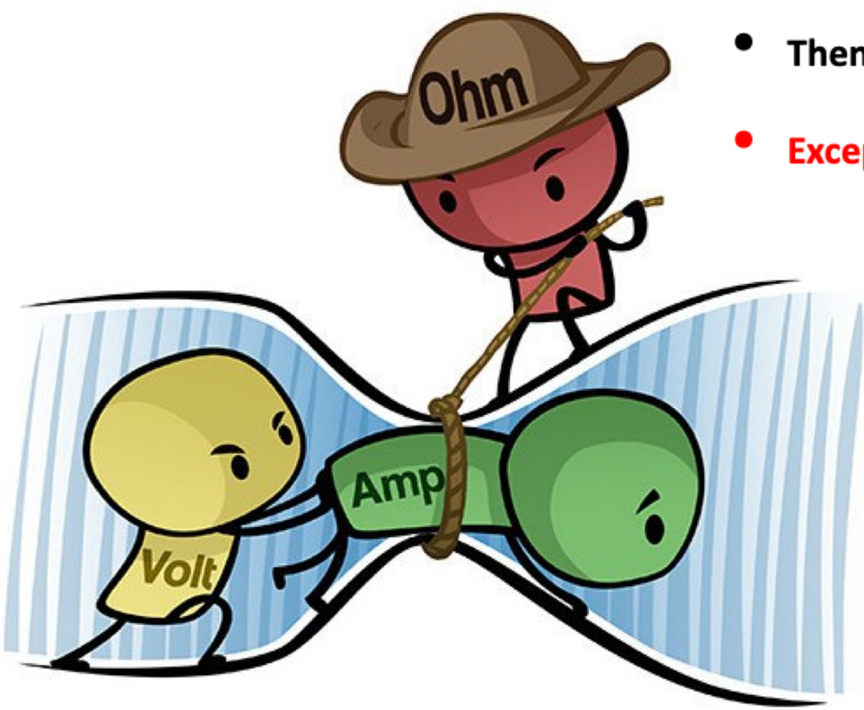
Last lecture:

- We established how measurements are performed
  - Define experiment, consistent conditions, complete data set
- Defined error / uncertainty
- Talked about the rounding conventions for this course

$$x = \bar{x} \pm \sigma_x$$

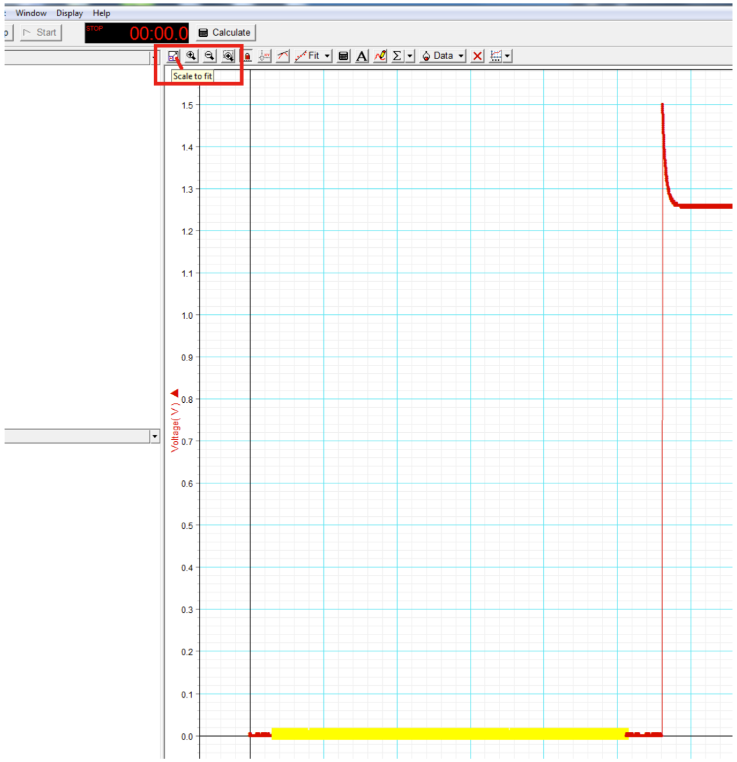
- **Round the error to one *significant figure***
- **Then round the mean to that *digit, , e.g. 5.6+/-0.2***
- **Exception: if the significant digit is "1", add one more digit, e.g. 5.66+/-0.13**

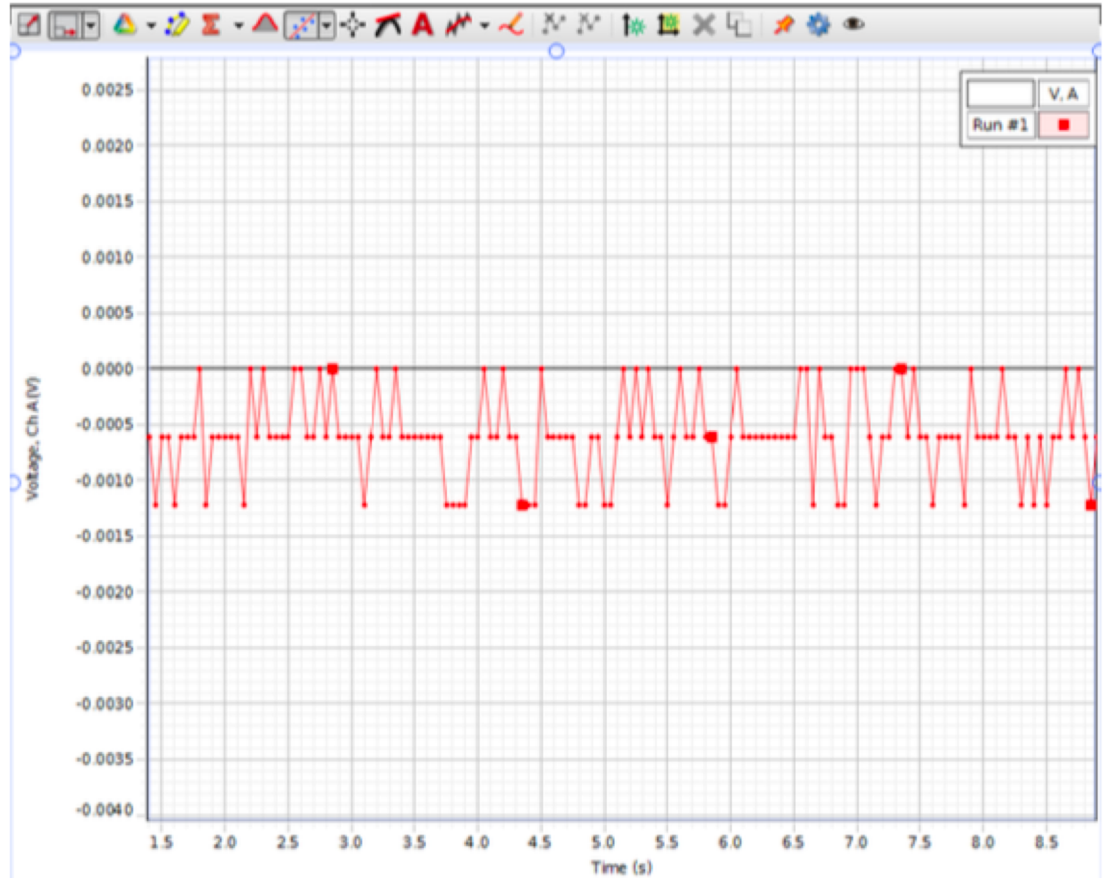


Lecture 2:
- Errors
- Error propagation
- Ohm's Law

Today

- Debrief on Lab 0
  - What fitting does for us
  - How we design experiments and fits to do science
- We examine how errors factor into calculations
  - **Errors**
  - **Error Propagation** formula
- Prepare for Lab 1
  - Resistors
  - Ohm's Law
  - Resistivity
- How fits will factor in to the lab 1 analysis





Lab 0

- We measure a quantity by sampling it many times
  - We average those measurements and compute uncertainty with standard deviation
- Our measurements are **rounded to reflect that error obscures our precision**
  - Round error to one significant figure
  - Round mean to the digit of error
- Example from Lab 0 (and every lab afterwards)

Error Propagation

$$x = \bar{x} \pm \sigma_x$$

Almost always, the scientific result is based on **interpretation** of a measurement

→ The measurement has uncertainty

→ The interpretation includes calculations, often that involve multiple measurements, each with its own uncertainty

Example: uncertainty in subtracting two variables

- Suppose I measure start and stop time as t_1 and t_2:

$$t_1 = 1.2 \pm 0.2\ s \quad \delta t_1 = 0.2\ s$$
$$t_2 = 54.6 \pm 0.2\ s \quad \delta t_2 = 0.2\ s$$

and want to know the difference between the two:

$$\mathrm{T} = t_2 - t_1 = 53.4 \pm ?\ s$$

$$\delta T = ?$$

Zoom practice: “Error in substraction”

What is the error associated with T?

A) 0.4 s

B) 0.28 s

C) 0.04 s

D) 0.3 s

Error Propagation formula

- Suppose we are calculating $f = f(x, y, z, \dots)$, a function of many variables, all of which have uncertainty.

- We need to know how a little change in the value of $x, y \dots$ affects the value of f

- This is characterized by **the partial derivative**.

For a single variable:

$$\delta f = \frac{\partial f}{\partial x}\delta x$$

For multiple variables that have independent, random uncertainty:

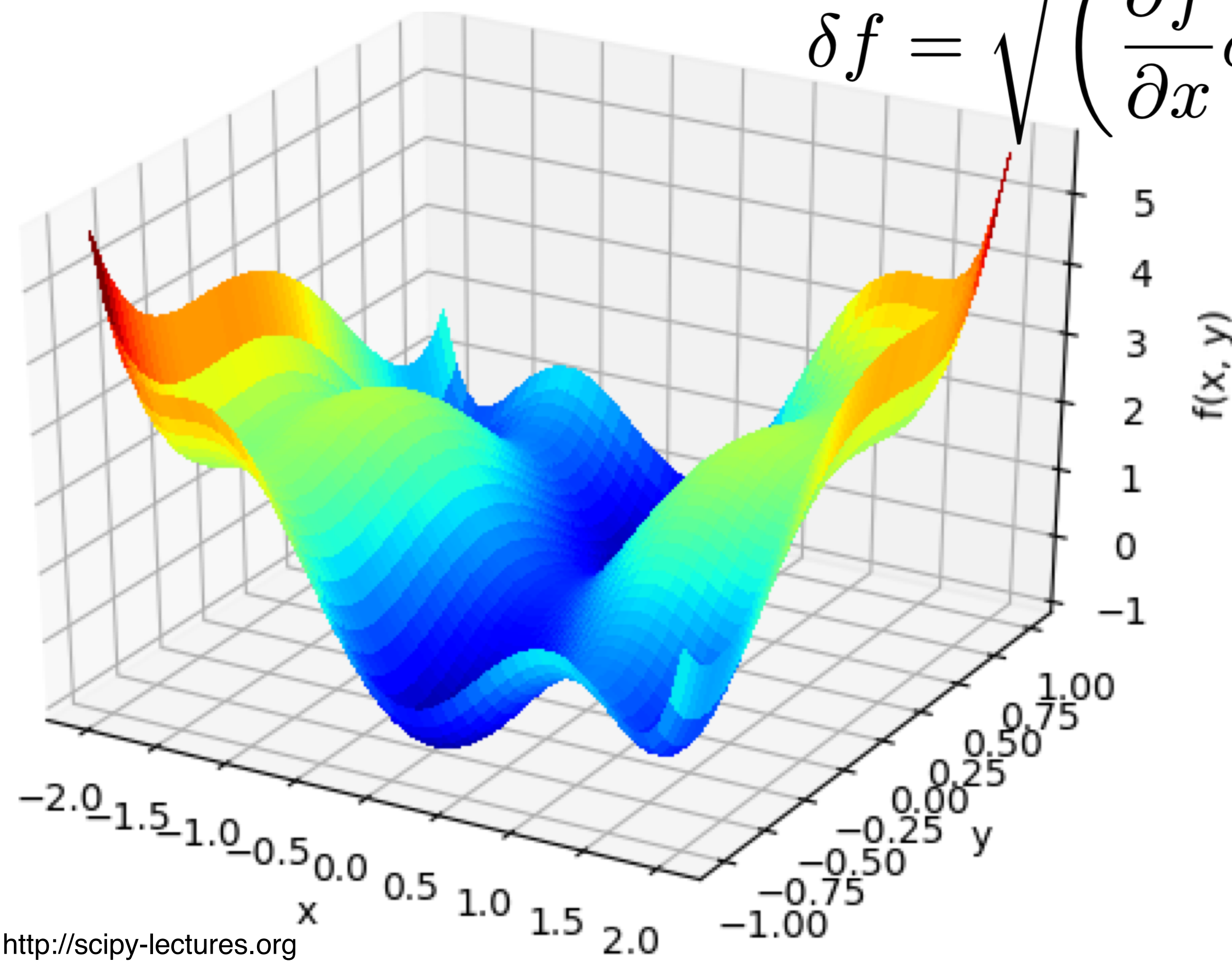
$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 + \left(\frac{\partial f}{\partial z}\delta z\right)^2 + \dots}$$

https://en.wikipedia.org/wiki/Propagation_of_uncertainty

For example, for a function of two variables:

$$f = f(x, y)$$

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2}$$



http://scipy-lectures.org

Error of a sum (or subtraction)

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 + \left(\frac{\partial f}{\partial z}\delta z\right)^2 + \dots}$$

S_x, S_y, S_u are the errors of x, y, u

$$u = x + y$$

$$s_u = \sqrt{\left(\frac{\partial u}{\partial x}\right)^2 s_x{}^2 + \left(\frac{\partial u}{\partial y}\right)^2 s_y{}^2}$$

$$= \sqrt{(1)^2 s_x{}^2 + (1)^2 s_y{}^2}$$

$$= \sqrt{s_x{}^2 + s_y{}^2}$$

$$u = x - y$$

$$S_u = \sqrt{S_x^2 + S_y^2}$$

Back to our example: uncertainty in subtracting two variables

- Suppose I measure start and stop time as t_1 and t_2:

$$t_1 = 1.2 \pm 0.2\ s \quad \delta t_1 = 0.2\ s$$
$$t_2 = 54.6 \pm 0.2\ s \quad \delta t_2 = 0.2\ s$$

and want to know the difference between the two:

$$\mathrm{T} = t_2 - t_1 = 53.4 \pm ?\ s$$

$$\delta T = ?$$

$$u = x - y$$

$$\delta T = \sqrt{0.2^2 + 0.2^2} = 0.28$$

$$S_u = \sqrt{S_x^2 + S_y^2}$$

$$T = 53.4 \pm 0.3\ \mathrm{s}$$

Error of a product

$$u = x \cdot y$$

$$s_u = \sqrt{\left(\frac{\partial u}{\partial x}\right)^2 s_x{}^2 + \left(\frac{\partial u}{\partial y}\right)^2 s_y{}^2}$$

$$= \sqrt{(y)^2 s_x{}^2 + (x)^2 s_y{}^2}$$

$$= \sqrt{y^2 s_x{}^2 + x^2 s_y{}^2}$$

$$= u\sqrt{\frac{s_x{}^2}{x^2} + \frac{s_y{}^2}{y^2}}$$

Relative Uncertainty

$$\sigma_x = s_x/x$$

$$\sigma_y = s_y/y$$

$$\sigma_u = s_u/u$$

$$\sigma_u = \sqrt{\sigma_x^2 + \sigma_y^2}$$

$$\delta_u = u\sigma_u$$

$$w = 0.59 \pm 0.03m$$

$$A = lw$$

$$l = 1.36 \pm 0.05m$$

Error Propagation

$$\delta_A = ?$$

$$\sigma_A = \delta_A / A$$

$$\sigma_A = \sqrt{\sigma_l^2 + \sigma_w^2} = \sqrt{(0.03/0.59)^2 + (0.05/1.36)^2} = 0.0627$$

$$\delta_A = A \cdot \sigma_A = 0.802 \times 0.0627 = 0.05$$

$$A = 0.80 \pm 0.05m^2$$

Summary of Error Propagation

- $f = f(x, y, z, \ldots)$, a function of many variables, all of which have uncertainty.
- For multiple variables that have independent, random uncertainty:

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 + \left(\frac{\partial f}{\partial z}\delta z\right)^2 + \ldots}$$

Zoom Exercise: "Error in Y^n"

Practice: what is the error of Y if $Y = X^n$, and X has an error of δx?

A) $\delta y = n\,\delta x$

B) $\frac{\delta y}{y} = n\,\delta x/x$

C) $\delta y = (n-1)\delta x$

D) $\frac{\delta y}{y} = (n-1)\frac{\delta x}{x}$

Summary of Error Propagation

Practice: what is the error of Y if Y = X^n, and X has an error of δx?

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 + \left(\frac{\partial f}{\partial z}\delta z\right)^2 + \ldots}$$

$$\delta y = \sqrt{[(nx^{n-1})\delta x]^2} = nx^{n-1}\delta x$$

$$y = x^n$$

$$\delta y / y = n\delta x / x$$

A) $\delta y = n\,\delta x$

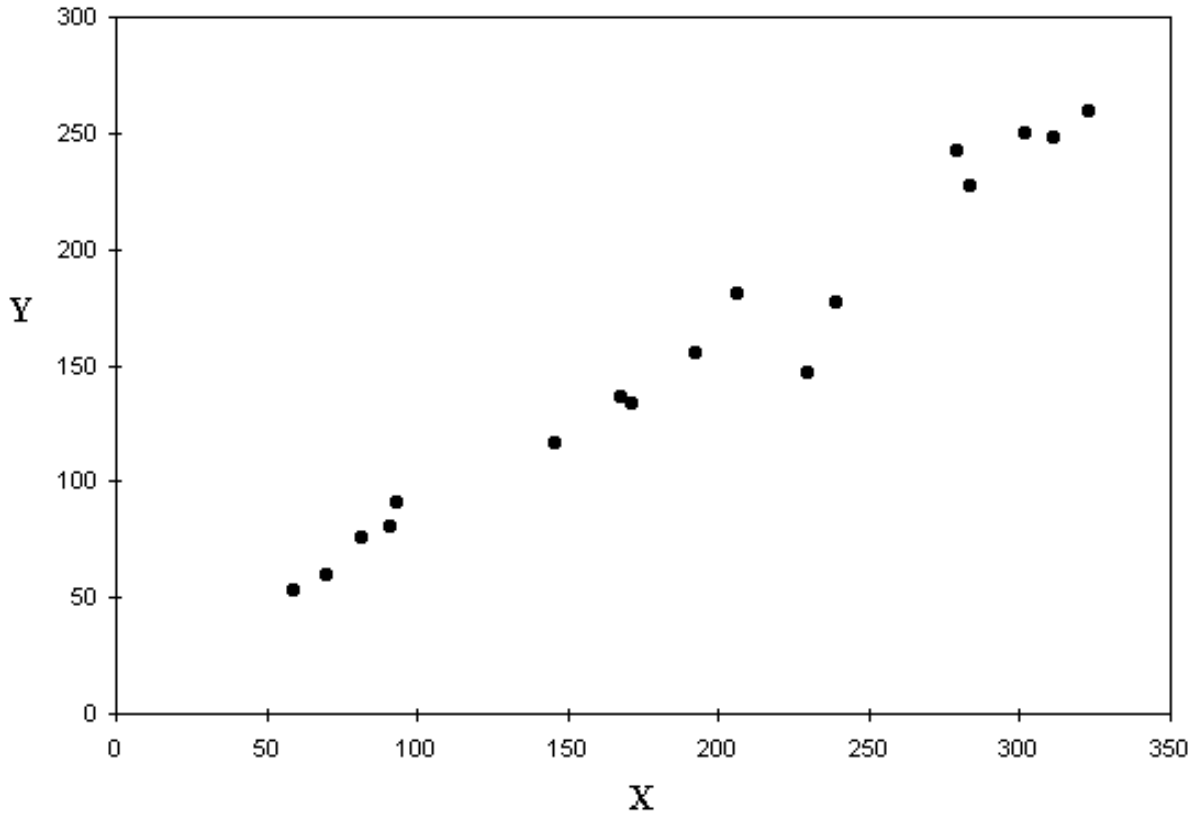
B) $\frac{\delta y}{y} = n\,\delta x/x$

C) $\delta y = (n-1)\delta x$

D) $\frac{\delta y}{y} = (n-1)\,\frac{\delta x}{x}$

Fitting data

Not to make it look pretty, but to compare data to a model → verify the model!



Fitting data

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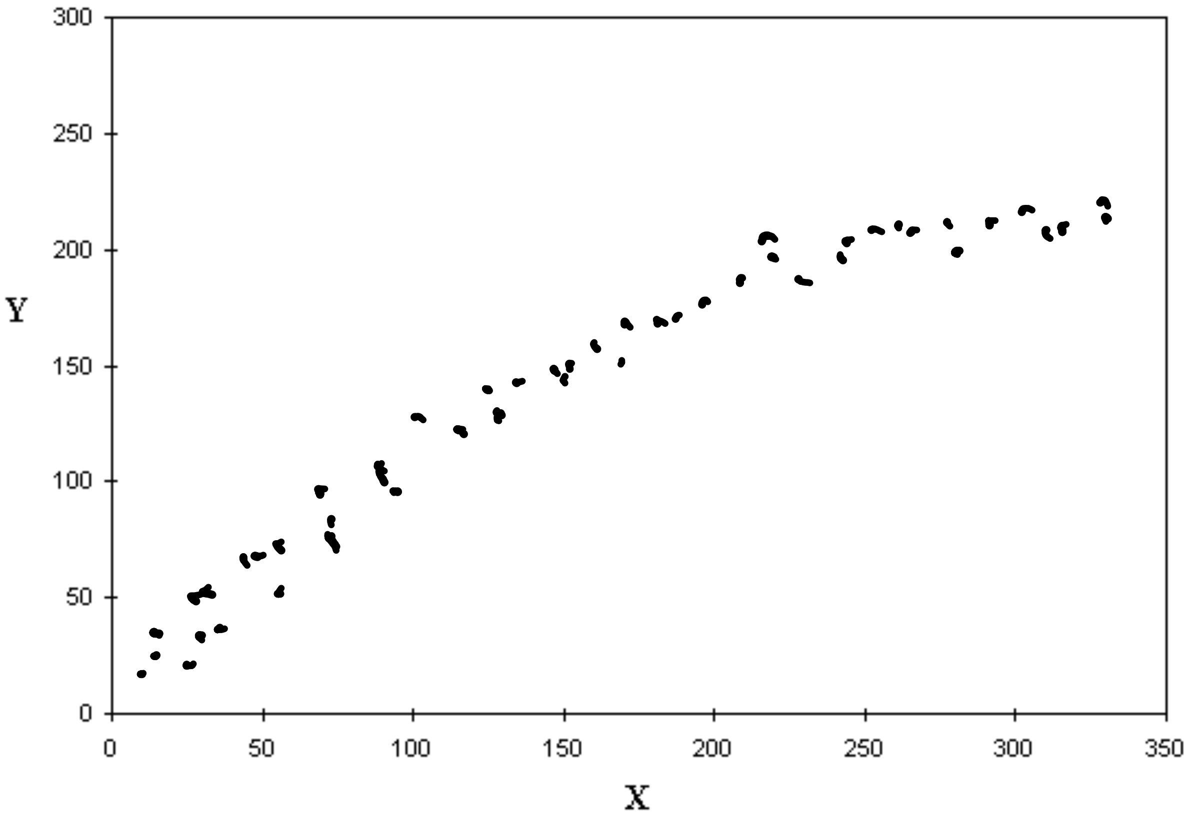
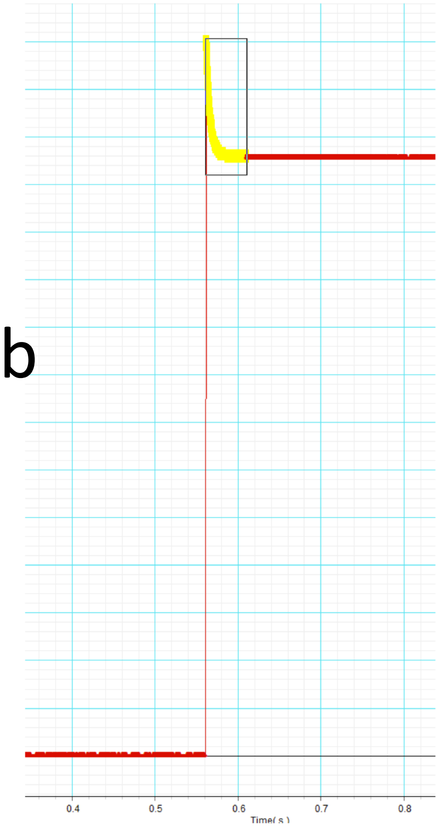


Fitting data → understanding data

$$y = Ae^{-\frac{t}{\tau}} + B$$



- What happened in Lab 0?
  - You were given some data set
  - You exported it and fit some curve to it in Matlab
- Why we did this:
  - We have some quantity we want to figure out

$$C = ?$$

  - We have theory which describes how it should behave

$$V = V_0 e^{-t/RC} + V_{ss}$$

Fitting data

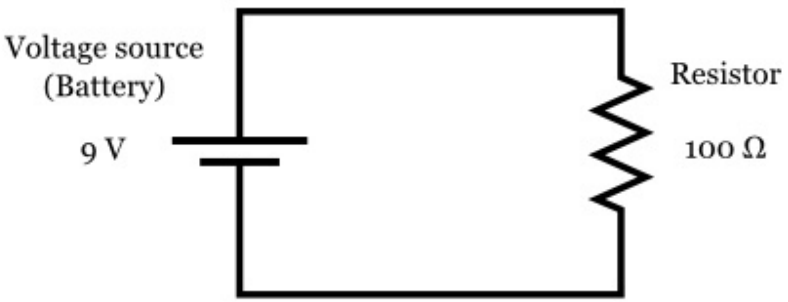
$$A = V_0$$
$$\tau = RC$$
$$B = V_{ss}$$

- We can now compare our fit result to the theory

$$y = Ae^{-\frac{t}{\tau}} + B$$
$$V = V_0 e^{-t/RC} + V_{ss}$$

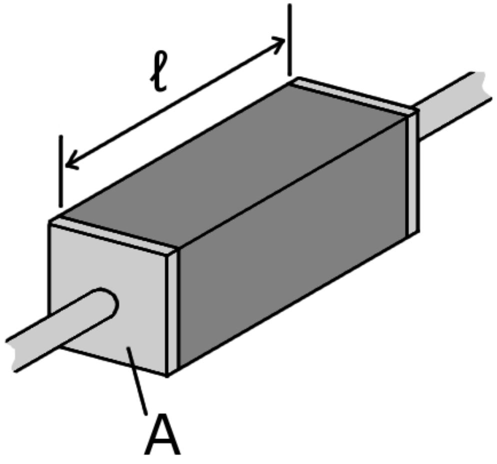
- Fitting is the best way to treat a large dataset
  - Rather than using individual points to calculate our values
  - Our Matlab fits give us uncertainty nicely

Lab 1: Ohm's Law



- Studying the most basic circuit element: Resistors
- Good resistors obey Ohm's Law

$$V = IR$$



- What makes a resistor resist current?

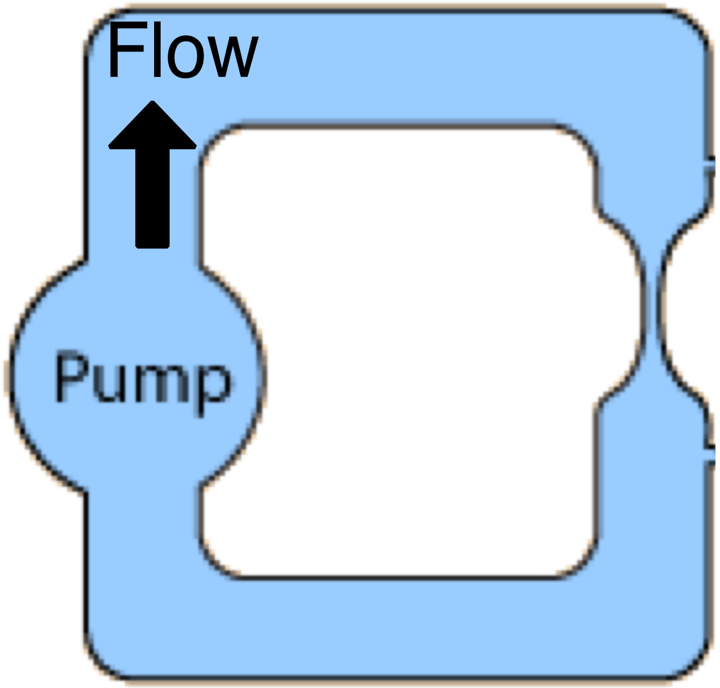
$$R = \rho \frac{L}{A}$$

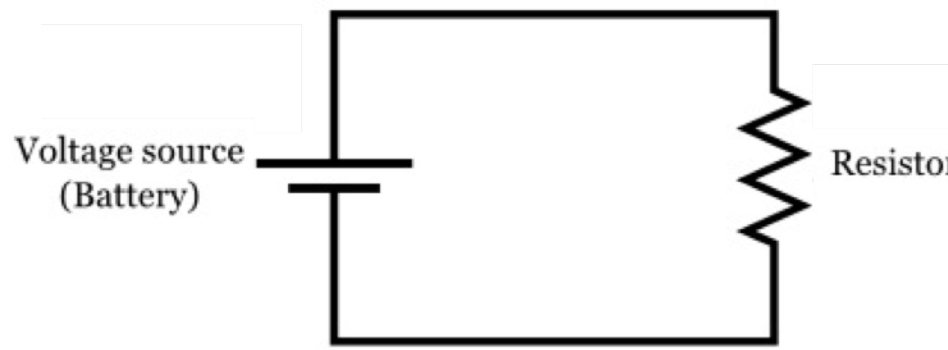
- Learn about how to measure current and voltage
    - By changing the circuit as little as possible

The main players in Physics 2CL

- Voltage (electric potential)
- Current
- Resistance
- Inductance (magnetic fields)
- Capacitance (electric fields)

Resistance





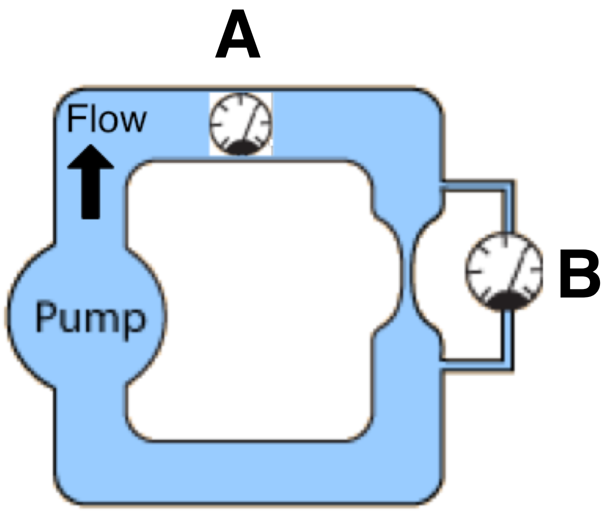
$$R = \rho \frac{L}{A}$$

Measuring Current

- Current is the flow of charge, measured by an ammeter, named from Ampere - Meter.

Zoom poll

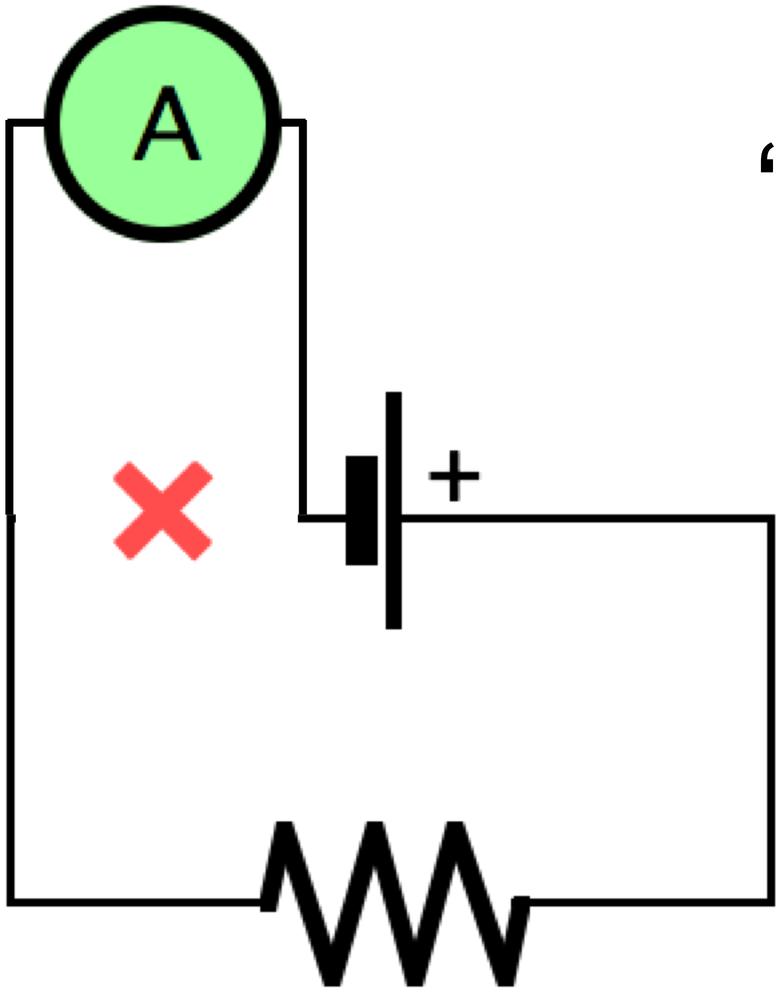
If I want to measure the flow of liquid through resistance,
where do I put my meter?
A) **A**
B) **B**



- The ammeter is connected in the circuit in series with the element whose current you would like to measure.

- We want the resistance of the ammeter to be very small, so it would not affect the existing circuit

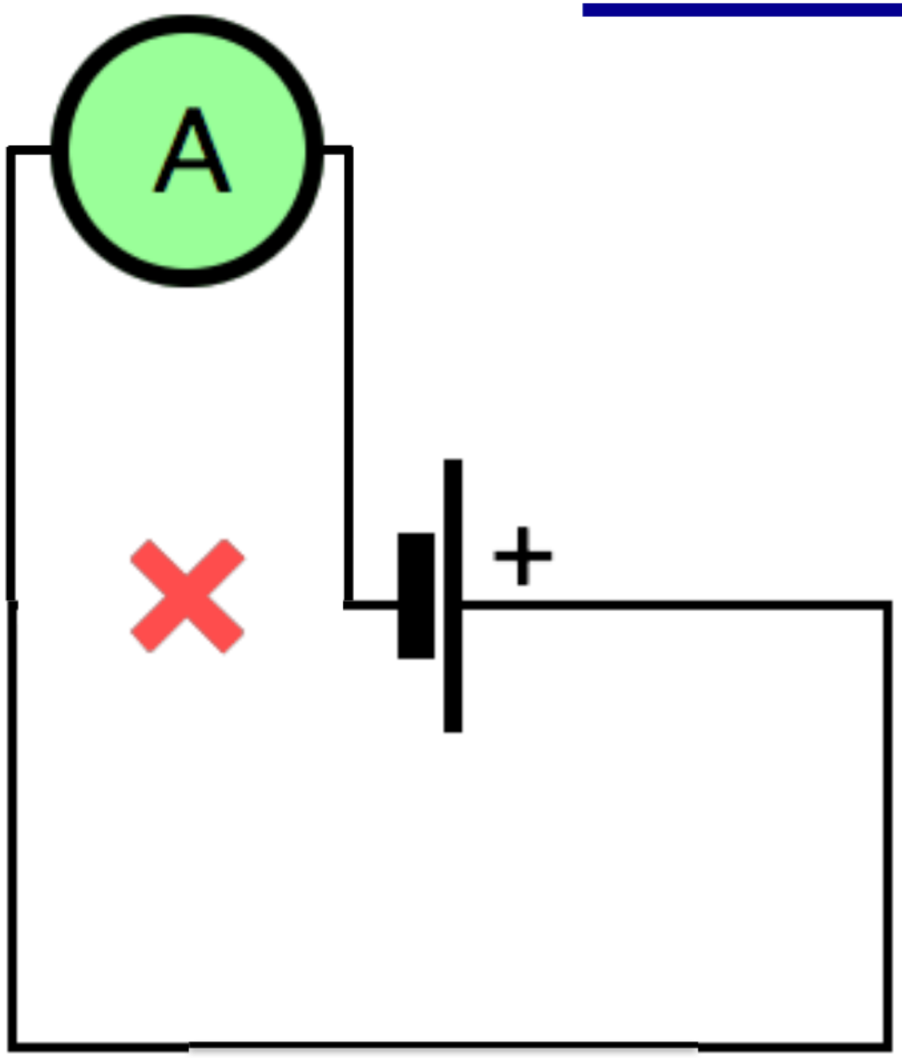
Ammeter



"Break open"
to detect flow

Warning: NEVER connect the Ammeter directly to a battery WITHOUT a resistor!

Ammeter



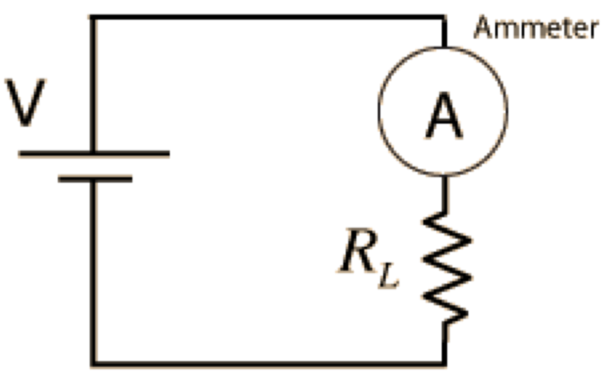
Warning: NEVER connect the Ammeter directly to a battery WITHOUT a resistor!

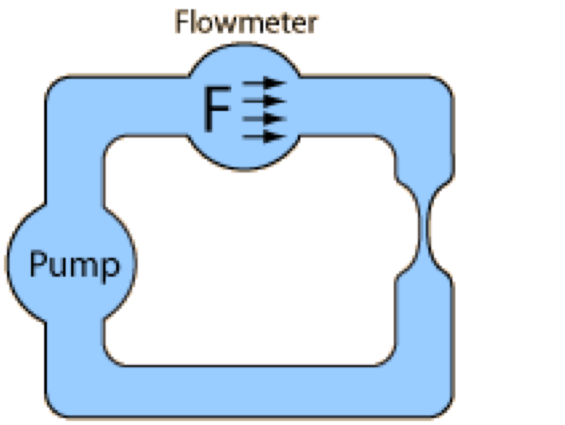
Ammeter

Why in series?

Analogy to water flow circuit

→ want to measure the flow that makes it to the element of interest.





hyperphysics.org

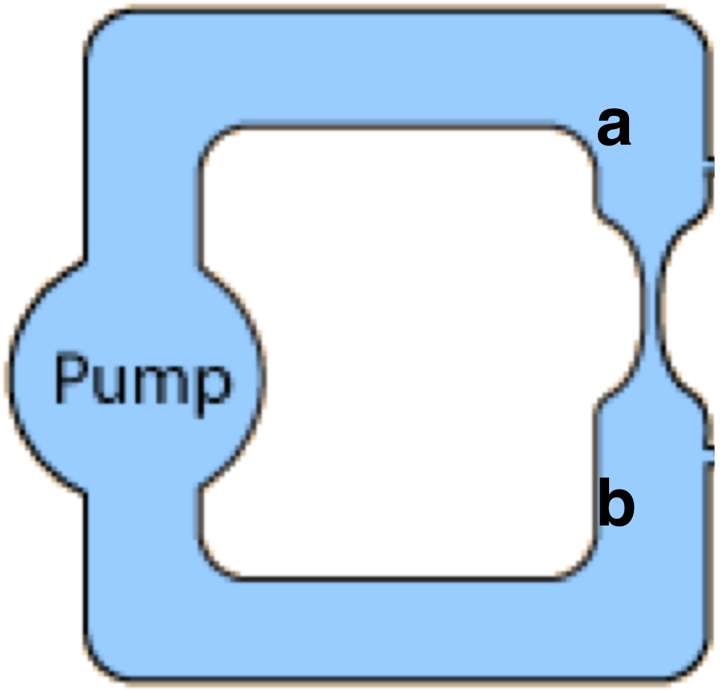
Measuring Voltage

- Voltage = electrical potential difference between two points
- A voltmeter is used to measure voltage across an element

Zoom Poll

→ What is the analog of voltage in flow?

a) temperature difference b/w **a** and **b**
b) flow difference b/w **a** and **b**
c) pressure difference b/w **a** and **b**



Voltmeter

- A voltmeter is used to measure electric potential difference (or voltage).

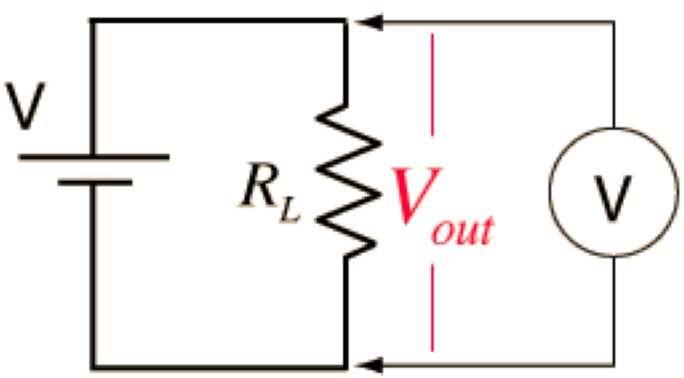
- The voltmeter is connected in the circuit in **parallel** with the element whose voltage (potential difference) you would like to measure.

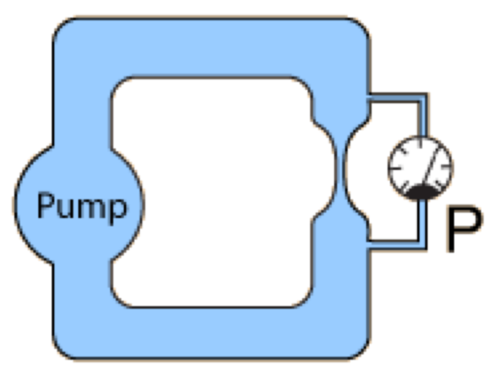
Why in parallel?

Analogy to water flow circuit = pressure difference

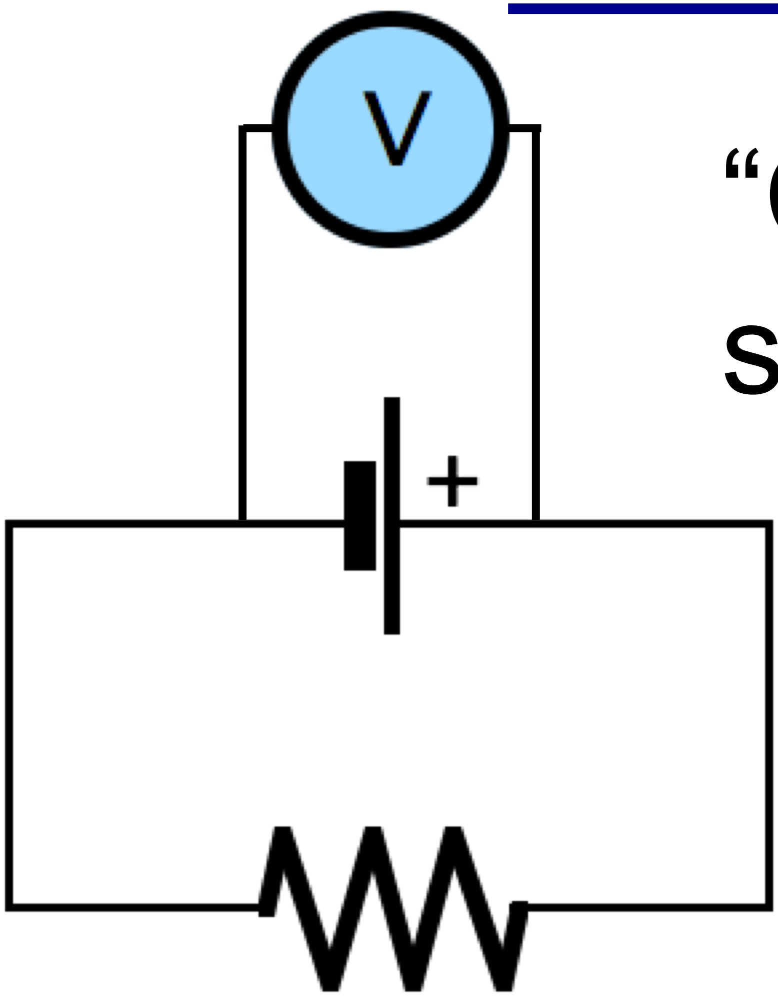
→ want to measure the pressure difference across the element of interest.

→ The resistance in a Voltmeter is usually very large, so it doesn't draw any current and affect the circuit

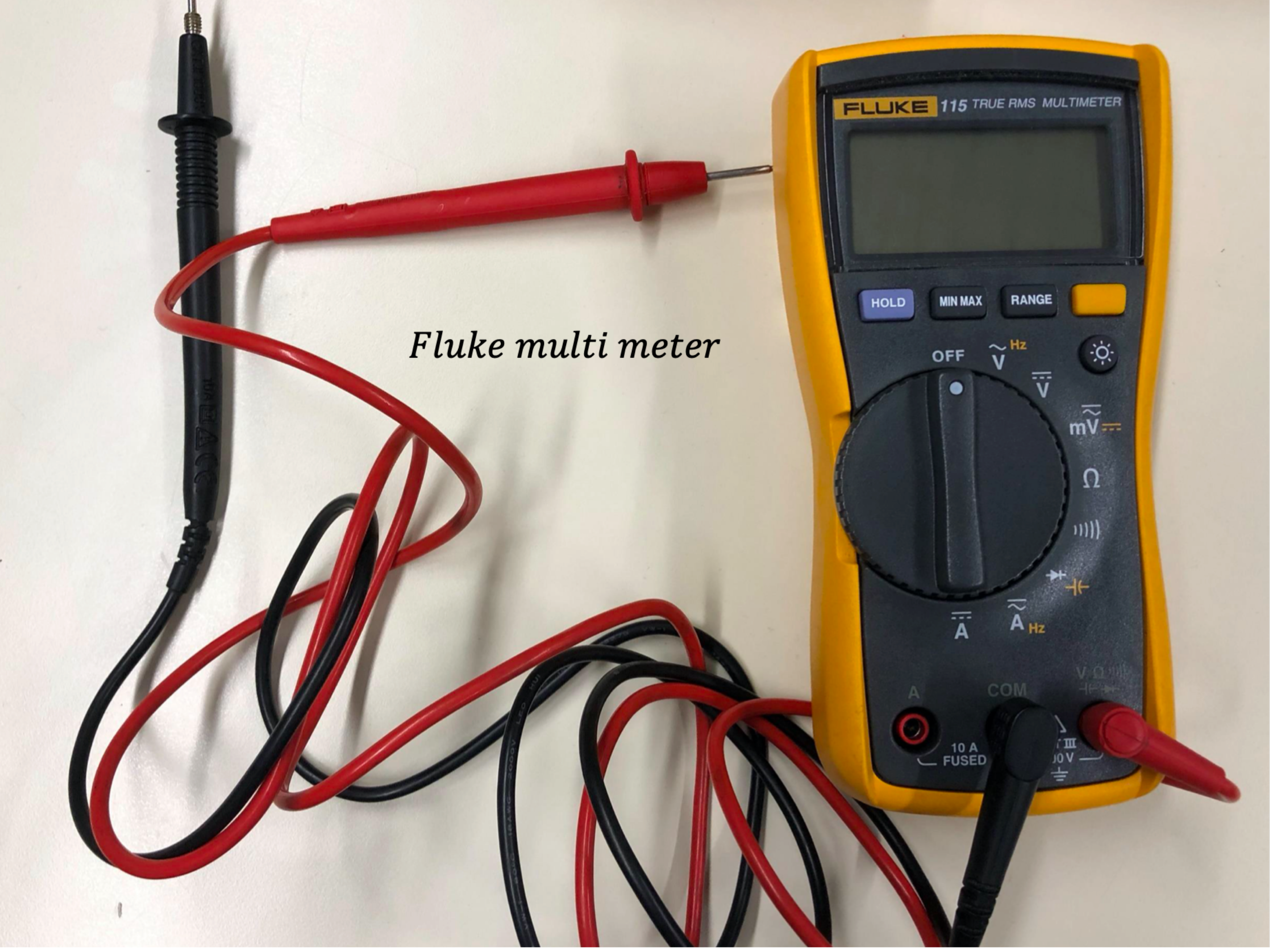




Voltmeter



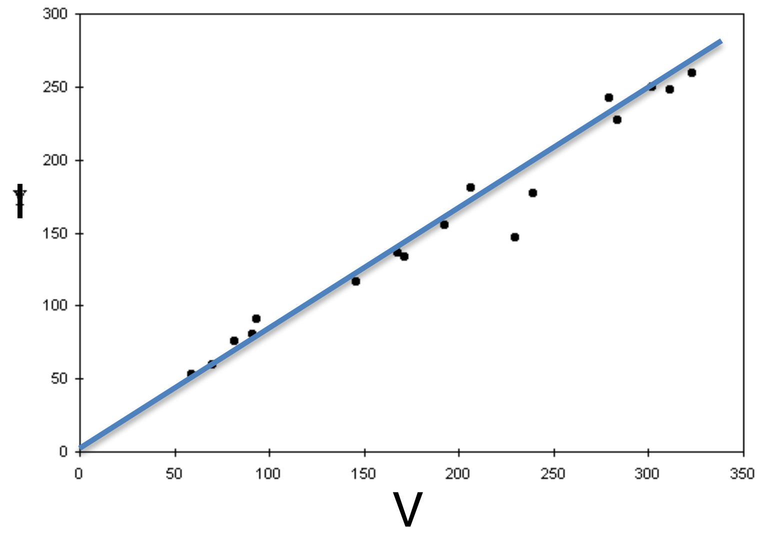
"Connect on both sides" of element



Fluke multi meter

$$y = f(x)$$
$$y = mx + b$$
$$I = \frac{1}{R}V$$

$$R = \frac{1}{m}$$



Lab 1 Example

- I want to know the resistance of some resistor

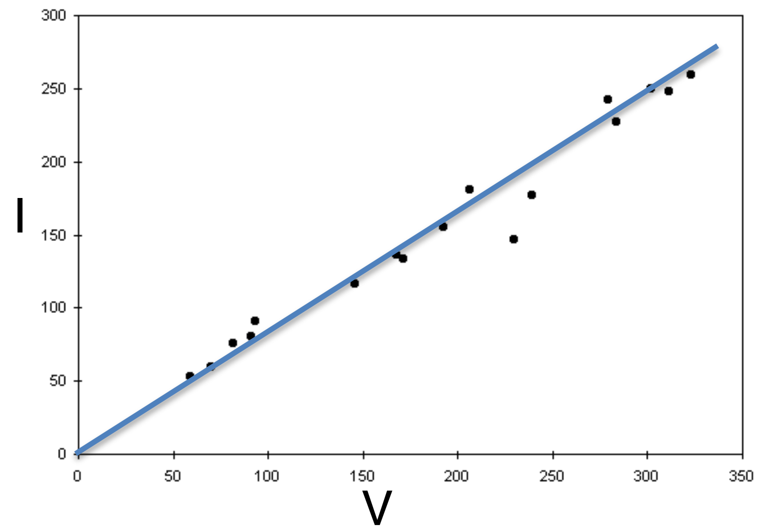
$$R = ?$$

- I "can't" measure it directly but I can measure voltages (*V*) and currents (*I*)
  - I have a theory (**Ohm's Law**) which relates *V*, *I*, and *R*

$$V = IR$$

- I'll collect several voltage-current pairs
  – Rather than use individual points I'll fit a function to the set

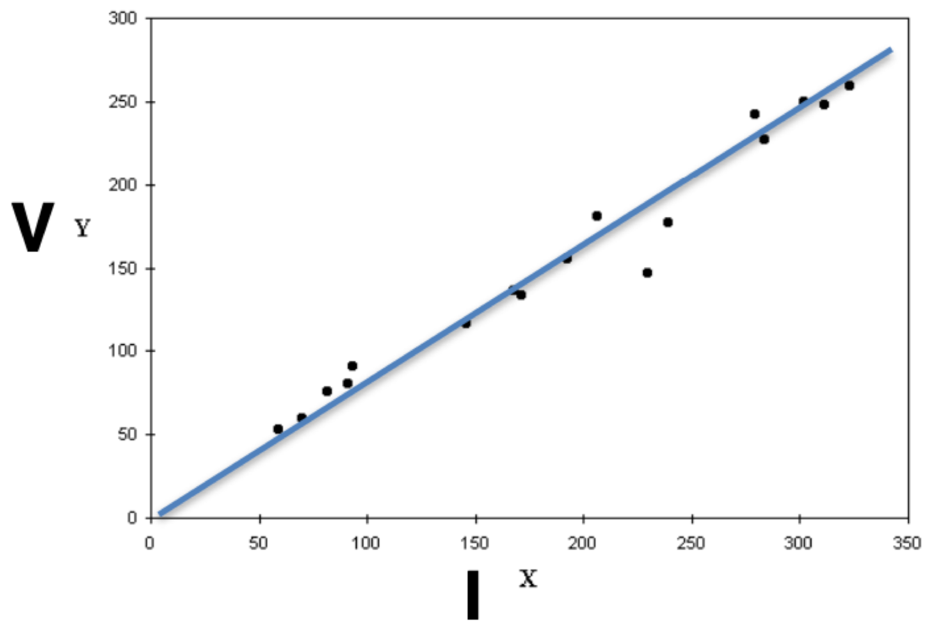
$$\delta R = \sqrt{\left(\frac{dR}{dm}\delta m\right)^2}$$



In Summary:

- We are going to perform an experiment to determine some quantity $R = ?$
- We know the theory which describes the behavior of that quantity $V = IR$
- Collect a dataset to analyze $I = \frac{1}{R}V$
- Decide which function to fit to that data
  - Should reflect the theoretical behavior $y = mx + b$
- Compare the fit and the theory to calculate the desired quantity
  - Calculate using error propagation! $R = \frac{1}{m}$

Another way to plot the data



$$V = IR$$

$$y = kx + y_0$$

$$R = k$$

$$\delta R = \delta k$$

Next lecture

- Interpreting error
  - Probabilistic interpretation of error
  - t-score, percentages from t-scores
- Capacitors
  - Basic definition, parallel-plate capacitor
  - Behavior in circuits, RC circuits

Assignments and reminders

- Read through the 2CL Lab 1 manual and watch the lab 1 video
- Turn in report lab 1 next Friday
- Your lab 0 report and HW#1 might be returned next week

- Understand the function of Ammeter, Voltmeter
- Understand the series and parallel connections

- Turn in the HW#2 this Friday
- Zoom office hours (links on Canvas):

Instructor: Monday 10:50 - 11:50 AM, stay on this Zoom session

Lead TA Coordinator: Thursday 3-4 pm